

Warm-up

Find the pattern and write the next three terms.

1. 2, 4, 6, 8, 10, 12 , 14 , 16 pattern:
Add 2

2. 1, 4, 9, 16, 25, 36 , 49 , 64 pattern:
Perfect squares
/ increasing odds

3. -2, 4, -8, 16, -32 , 64 , -128 pattern:
mult. by -2

4. $1/2$, 1, $3/2$, 2 , $5/2$, 3 , $7/2$ pattern:
Add $1/2$

Unit 8

Probability and Statistics

Sequences and Series

- Ordered list of numbers

- Notation: a_n

- Sum of a sequence

- Notation: S_n or \sum

Can be finite (1, 2, 3) or infinite (1, 2, 3, 4, ...)

Sequences and Series

2 types:

Arithmetic: Pattern progresses by adding/subtracting (Common difference)

Geometric: Pattern progresses by multiplying/dividing (Common ratio)

Equation for Arithmetic Sequence:

$$a_n = a_1 + (n-1)d$$

Finding
a specific
term

a_n is the n^{th} term of the sequence

a_1 is the first term

d is the common difference

what you are
adding/subtracting

Examples

1. Write the ~~next~~ ^{17th term} term of the sequence. Then write a rule for the ~~nth~~ term.

a. $-4, -3, -2, -1, \dots$

$$a_n = a_1 + (n-1)d$$

$$a_n = -4 + (n-1)(1) \quad n^{\text{th}} \text{ term rule}$$

$$a_{17} = -4 + (17-1)(1)$$

$$a_{17} = 12$$

c. $0, 1, 2, 3, \dots$

d. $5, 1, -3, -7, \dots$

$6, 9, 12, 15$

General rule : $a_n = 6 + (n-1)(3)$

$$a_{17} = 54$$

2.

a. Write a rule for the n th term of the sequence 32, 47, 62, 77, Then find a_{12} .

b. Write a rule for the n th term of the sequence $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, Then find a_{12} .

$$\begin{matrix} n^{\text{th}} \\ \text{rule} \end{matrix} : a_n = \frac{1}{2} + (n-1) \frac{1}{2}$$

$$a_{12} = 6$$

Arithmetic Series

Equation for arithmetic series:

$$S_n = n \left(\frac{a_1 + a_n}{2} \right)$$

Sum
of
a
specific
number
of terms

S_n is the sum of the first n terms of the series

Examples subtracting 2

1. Arithmetic series: $20+18+16+14+\dots$

Find the sum of the first 25 terms.

$$a_{25} = 20 + (25-1)(-2) = -28$$

$$S_{25} = n \left(\frac{a_1 + a_n}{2} \right) = 25 \left(\frac{20 + (-28)}{2} \right) = -100$$

2. Arithmetic series $100+110+120+130+\dots$

Find the sum of the first 18 terms

$$a_{18} = 100 + (18-1)(10) = 270$$

$$S_{18} = 18 \left(\frac{100 + 270}{2} \right) = 3,330$$

Sigma Notation of a Series

last
number
you plug in

Sum \rightarrow
$$\sum_{n=1}^5 \underbrace{(1 + 5n)}_{\text{pattern}}$$

first number
you plug in to the pattern

Examples

$$1. \sum_{n=1}^5 (1 + 5n) = 80$$

$$a_1 = 6$$

$$a_2 = 11$$

$$a_3 = 16$$

$$a_4 = 21$$

$$a_5 = 26$$

$$\begin{array}{r} 6 \\ 11 \\ 16 \\ 21 \\ 26 \\ \hline 80 \end{array}$$

$$2. \sum_{n=3}^7 (n^2 + 4) = 155$$

$$a_3 = 13$$

$$a_4 = 20$$

$$a_5 = 29$$

$$a_6 = 40$$

$$a_7 = 53$$

$$\hline 155$$

You Try

$$3. \sum_{n=1}^5 (4n^2 + 1) = 225$$

$$a_1 = 5$$

$$a_2 = 17$$

$$a_3 = 37$$

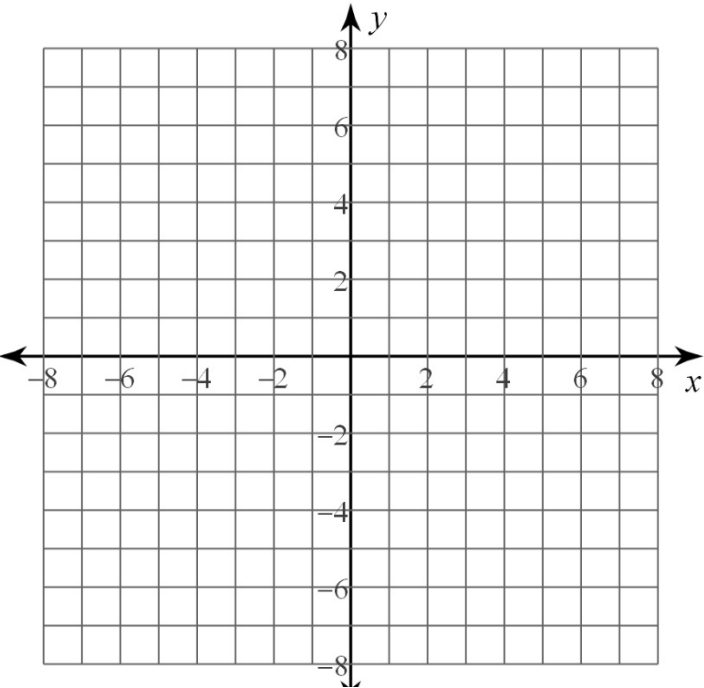
$$a_4 = 65$$

$$a_5 = 101$$

$$\underline{225}$$

4. $\sum_{n=4}^8 (50 - n)$

$\log_2 (x - 3) + 1$



$$-\log_2 x - 4$$

